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# Polarization Enhanced Production in a Membrane Reactor

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# POLARIZATION ENHANCED PRODUCTION IN A MEMBRANE REACTOR

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ABSTRACT

The membrane-based catalytic reactor with "free" catalyst has been modeled simply and studied by the authors on a theoretical basis. The hypothesis supporting an advantage of the membrane reactor is two-fold: a) perfect rejection by the membrane and small reject fraction concentrates the catalyst highly in a steady-state continuous flow reactor (SSCFR) and b) the membrane reactor produces a filtered, high quality product. Of further advantage is the potential for concentrating the catalyst highly near the membrane in a thin diffusion dependent zone wherein the reacting substrate is also concentrated. The conjunction of concentrated catalyst and substrate leads to less inhibition of the reaction. The model involves conventional diffusional theory, simple membrane characterization of uniform flux, and Michaelis-Menten kinetics.

## INTRODUCTION

Membrane Reactors have been conceived by investigators for a considerable period. Cho et al. (1)

have indicated potential for continuous reaction and separation in a vessel. The state of the art typical of the times of study have been presented in several papers [Spalding (2), Kearns (3), Michaels and Matson (4)]. In one early investigation Drioli and Scardi (5) placed both enzyme and substrate in a pressurized cell containing a membrane surface. The investigation was followed by a number of studies in which the enzyme was immobilized on the membrane surface in such a way that the fluid being processed would bring the reactants to the site of membrane and the filtration would immediately follow. Gruesbeck and Rase (6) presented an insolubilized enzyme study for sugar production. Drioli et al. (7) presented results of some reactors having immobilized *calderiella acidophila* for high temperature reactions. Alfani et al. (8) have reported attempts to convert cellulose for potential energy applications. Thomas, McKamy and Spencer (9) have reported production of sugar in a membrane-based immobilized enzyme reactor.



## THEORY

A membrane reactor in the sense of this paper acts typically as a device encompassing a fluid undergoing reaction and selectively sieves the reactants or products from the reacting mixture. Thus its use is three fold. (1) It acts as a container for the reaction to take place; (2) it selectively passes or retains the products and /or reactants and thus effectively acts as a separation tool, and (3) it concentrates the reactants and the catalysts by the membrane action to enhance the reaction rates. It is our aim herein to study the net enhancement of production rates by the concentration

polarization of the reactants brought about by the membrane action.

Generally there are two types of reactors: (1) continuous reactors and (2) batch reactors. A membrane reactor is readily adapted to the continuous type reaction. It is anticipated that a conventional continuous reactor will feed the membrane reactor resulting in a combination of a continuous and membrane reactor.

Enzyme as a catalyst is added to a prepared substrate stream (Figure 1) in an amount estimated to bring about a desired reaction. At a predetermined conversion fraction the flow is augmented with additional catalyst and fed into a membrane reactor. The membrane is presumably located in a recirculating stream. The permeated fluid primarily contains the product of reaction in a filtered form as the membrane is assumed to perfectly retain the reactant and the catalyst. The loop accepting the feed must reject a small stream to avoid collection of contaminants and partially reacted substrate. The reject stream contains minor, but potentially important, amounts of product and unreacted substrate. We consider the reactor to be a continuous reactor although the application may well have a batch or batch-like process producing nearly the same effect on a fluctuating basis.

Advantages of the concept are primarily in that the product is filtered as it is produced. The bulk phase reactor, wherein the reaction is essentially done at uniform bulk phase conditions, may afford certain advantages by itself. The first advantage is that the effect of concentrating the enzyme in the recirculating membrane loop adds to the rate of production. Because the reject flow is perhaps 0.04 of the incoming flow,

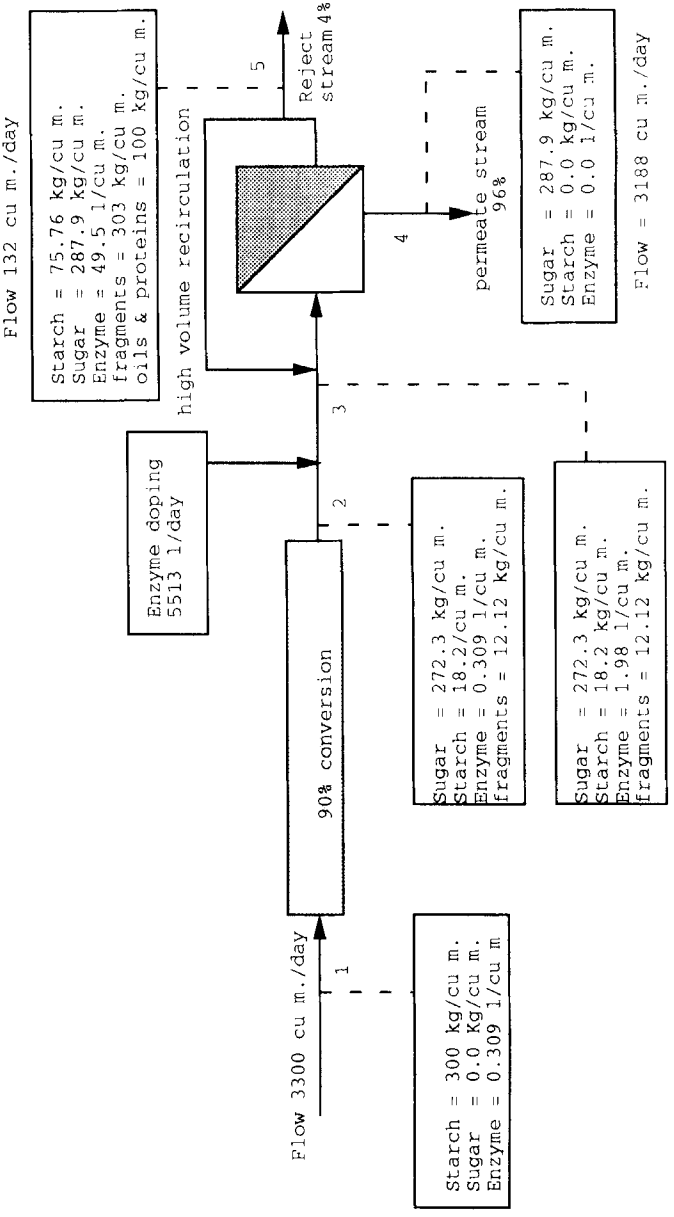


FIGURE 1. Material Balance for the Reactor.

and assuming the enzyme is completely rejected by the membrane, the enzyme becomes 25 times more concentrated in the loop than it was in the feed stream. So, in one hour of residence in the loop, the production is equivalent to 25 hours at the concentration of the continuous reactor. Also, the residence time within the loop is proportional to the volume per unit area and reciprocally on the flux. The residence time should be approximately one-half hour for common parameters. This means that the final one or perhaps two percent of the total reaction should be occurring in the membrane reactor and this in a short time. If there is some addition of enzyme between the continuous reactor and the membrane reactor, the enzyme addition may potentially be controlled to yield a finished product of some specification by controlling, on a real-time basis, the rate of addition of enzyme. At present, the batch reaction is typically surveyed at a time of 2/3 of the total reaction time and a decision is made whether to add additional enzyme. The end of the process can additionally be delayed slightly, but the production pressure on the tank volume may cause premature stopping of the batch reaction. The point is that 8 or 12 hours is an excessively long time over which to control effectively the outcome of a particular reaction. One half hour is much more reasonable. In addition, it is possible that the naturally occurring concentration polarization of the substrate and enzyme adjacent to the membrane may cause accelerated reactions compared to the bulk phase. This possibility is confirmed in the calculation of Datta (10) as a net positive effect. It is not clear whether the effect, because it occurs in such a small zone of thickness, can be enough to affect the production of a hypothetical reactor materially.

To determine whether the advantage is of any consequence, a model system is employed of starch-enzyme-sugar common in the corn industry. Normally this reaction is accomplished in batches requiring from one to three days of reaction prior to filtration and subsequent purification steps. Herein we propose a continuous reactor of one day throughput followed by a membrane reactor having a flux estimated for the conditions and a characteristic volume per unit area (see Figure 1). Various enzyme loadings are injected into the feed and immediately before the membrane unit to accomplish the overall conversion. In this calculation the membrane unit is assumed to behave at a single bulk phase concentration state. This is termed a uniform state reactor design equivalent to a continuously stirred tank reactor. The purpose of this exercise is primarily to establish the range of conditions reasonably appropriate for the membrane unit. However the uniform state reactor design unit may be of some interest for economic study in itself. Using the set of bulk phase conditions, the conditions predicted in the membrane reactor with polarization will be computed to determine the amount of advantage possible to gain.



A glucoamylase commercially available from NOVO (12) is assumed. Data supplied as technical literature describes the conversion of starch to reducing sugars as dependent on doping and time. The sugar production data were fitted to the Michaelis-Menten equation:

$$\frac{dC_3}{dt} = \frac{k_1 C_1 C_2}{C_1 + K_m} \qquad (1)$$

Equation (1) relates to the production of substrate as:

$$\frac{dC_1}{dt} = -MWR\frac{dC_3}{dt} = -\frac{k_1 C_1 C_2 MWR}{C_1 + K_m} \quad (2)$$

Observations from these calculations are that the substrate inhibition index, $K_m$, is very large compared with the 30% by mass value of substrate initial concentration. Since the membrane calculations are at substrate concentrations much lower than the original level, the value of $K_m$ is assumed to be essentially infinite. Thus the $k_1$ rate constant is determined as $k_1 * MWR/K_m$ and no substrate rate leveling at high concentration occurs. Further, the NOVO (12) data for sugar production show an exponential decline in production that approaches an apparent asymptote of 96% equivalent reducing sugar units. Therefore we have modeled the process as one in which 4% of the original substrate (starch) is unavailable for conversion. The substrate declines according to the integral of (2):

$$1 - \frac{C_1}{C_{1,0}} = 1 - e^{\left(-k_1 \frac{MWR}{K_m} C_2 t\right)} \quad (3)$$

The statement made here is that 4% of the original starch becomes an intermediate fraction not available for conversion. This remaining 4% comprises starch fragments, proteins, fats and oils occurring naturally in corn starch. Since the permeate comprises solely sugar, the reject stream carries the residual starch, starch fragments, oils, proteins, fats and enzyme. The other 96% is reacted to product according to equation (3). Thus, except at small times,

$$1 - \frac{C_1}{C_{1,0}} = \frac{C_3}{C_{3full}} + 0.04 \quad (4)$$

Values for the reaction constants, consistent with the NOVO data, are $K_1 * MWR/K_m = 0.3788$. Here the units

supplied by NOVO are in terms of liters of standard-activity enzyme.

The batch reaction equation used to correlate the NOVO data may be applied to a continuous reactor by substitution of $\dot{V}/A_c x$ for $1/t$ in equations (1), (2) and (3). Therefore this equation can be expressed in terms of the length of the continuous flow reactor x, the volume flow rate $\dot{V}$ and cross section of this reactor $A_c$. The resulting equation with the value for $\lambda = k_1 * MWR/K_m$ expresses the concentration of substrate remaining at any position $x/x_{max}$ in the continuous reactor:

$$1 - \frac{C_1}{C_{1,0}} = 1 - e^{\left(-\lambda C_2 \frac{A_c x}{\dot{V}}\right)} \tag{5}$$

and for the product:

$$\frac{C_3}{C_{3,full}} = 0.96 - e^{\left(-\lambda C_2 \frac{A_c x}{\dot{V}}\right)} \tag{6}$$

The value of $C_{1,0}$ is taken as 300 kg/m$^3$ (30% by mass). It is the goal to produce $C_1 = 0.01 * C_{1,0}$ exiting the reactor with $C_3 = 0.95 * C_{3,full}$. Values of starch from 1% to 2.5% are selected to remain at the end of the continuous reactor. Hence $C_1/C_{1,0}$ ranges from 2% to 3.5%. These correspond to particular lengths of the continuous reactor.

When the substrate consumption is is $10^6$ kg/day with a concentration of 300 kg/m$^3$, the solvent passage is 3300 m$^3$/day. If the reject fraction is RF, only (1-RF) of the solvent passes through the membrane corresponding to membrane area $A_m = 3667$ m$^2$ for flux=$10^{-5}$ m/s. According to the design consideration $A_c x/\dot{V}$, the residence time, is 24 hours. From equations (5) or (6)

$C_2$ can be calculated. The concentration and total amount of enzyme requirement per day for the continuous reactor is estimated as follows:

| $C_1/C_{1,0}$ | $C_{3f}/C_3$ | $C_2$ (l/cu m. of solution) | $\dot{E}$ (liters/day) |
|---|---|---|---|
| 0.02 | 0.94 | 0.430 | 1420 |
| 0.025 | 0.935 | 0.406 | 1339 |
| 0.03 | 0.93 | 0.386 | 1273 |
| 0.035 | 0.925 | 0.369 | 1217 |

Here $C_{1,0}$ refers to the concentration $C_1$ entering the reactor at x=0.

The reaction rate depends on mass and thus on volume. Therefore the $V/A_m$ ratio of this reactor must be specified in addition to the flux through the membrane. Within the uniform state reactor the modified Michaelis-Menten equation holds with $K_m >> C_1$:

$$\frac{dn_1}{dt} = -\frac{k_1 C_1 C_2}{K_m} \times MWR \times V \tag{7}$$

The sugar concentration passing point 2 (Figure 1) is $C_{3f}$ and in the loop is $C_3$ at point 5. From a mass balance and assuming that (1-RF) of the solvent flows through the membrane we get:

$$\frac{dn_1}{dt} = -\frac{dn_3}{dt} \times MWR = -MWR \times (C_3 - C_{3f}) \times \frac{J \times A_m}{1-RF} \tag{8}$$

Solving for $C_2$ from Equation (7) and (8) and rearranging we get:

$$C_2 = \left(\frac{K_m}{k_1 \times MWR}\right) \times \frac{J}{(1-RF) \times \left(\frac{V}{A_m}\right)} \times \left(\frac{C_1 - C_{1,m}}{C_{1,m}}\right) \tag{9}$$

where $C_{1,m}$ refers to the mean concentration of substrate in the membrane reactor.

For a typical comparison the following values were assumed in the membrane reactor mode. The value for the reject flow rate (RF) is established by the concentration of the oils and protein content in the original feed together with the maximum values of concentration consistent with reliable operation. Experience suggests RF=4% is near the correct value. Values from 5% of the flow to as little as 1% certainly define the range. However the losses of product associated with this reject flow are significant and are recoverable at least to some extent.

The value of the target conversion is chosen to correspond to a common (high dextrose) industry stream of 95% Dextrose Equivalent. Other targets could be selected. The summary of the input variables are as follows.

| | |
|---|---|
| Permeate Flux J | = $10^{-5}$ $m^3/m^2$-sec |
| $V/A_m$ ratio | = 0.01 m |
| Membrane Area $A_m$ | = 3667 $m^2$ |
| Permeate Flow Rate | = 96% of Feed Flow Rate |
| Reject Fraction (RF) | = 4% of Feed Flow Rate |
| Total Flow Rate | = 3300 $m^3$/day |
| $C_{1,0}$ | = 300 $kg/m^3$ |
| $C_{1,m}$ | = 3 $kg/m^3$ |
| Residence Time($A_c x_{max}/\dot{V}$) | = 24 hours |
| $K_1$*MWR/$K_m$ (from NOVO) | = 0.3788 $m^3$ of soln./lit of enzyme-hr. |

The solution of $C_2$ from equation (9) for different conversion factors are tabulated below:

| $C_1/C_{1,0}$ | $C_{3f}/C_3$ | $\dot{E}$ in CR (lit/day) | $C_2$ in MR (l/cu m.) | $\dot{E}$ in MR (lit/day) | doping $\dot{E}$ (lit/day) |
|---|---|---|---|---|---|
| 0.02 | 0.94 | 1420 | 9.9 | 1306 | 0 |
| 0.025 | 0.935 | 1339 | 14.85 | 1960 | 621 |
| 0.03 | 0.93 | 1272 | 19.8 | 2614 | 1342 |
| 0.035 | 0.925 | 1217 | 24.75 | 3267 | 2050 |

It can be seen that the enzyme requirement for the membrane reactor is sometimes only that of the continuous reactor that is used upstream to partially degrade the starch. Otherwise it is necessary to dope an excess amount of enzyme between the continuous reactor and the membrane reactor. The amount of enzyme to be doped is shown in the above table. The schematic flow chart in Figure 1 depicts the bulk concentrations at different sections of the steady state continuous flow reactor (SSCFR).

The Enhanced Reactor



Because of the membrane action, which rejects both substrate and catalyst, the concentrations of these components is increased near the surface of the membrane. The reaction equation (1) discloses that the production rate should increase in proportion to both these polarized components. Because the rates increase and potentially dramatically, it is recognized that the membrane reactor could be more productive than the uniform state reactor just presented. Offsetting the increase is the realization that the diffusive zone is very thin and may be limited in its ability to affect the entire reactor.

Datta (10) has shown it is possible to expect an enhancement compared with an equal volume of uniform state reaction when the membrane is polarizing the fluid. Further, he showed that the productivity has a maximum with respect to the length of the membrane channel. The length of the optimal channel corresponds to the position where the substrate, despite being concentrated, actually becomes seriously depleted due to the action of the polarized enzyme. It is desired to evaluate, using the techniques developed, the SSCFR

under the conditions just advanced for the effect of polarization enhancement.

Summary of the Model

It has been assumed that the flux across the membrane is constant [v=-J] and the velocity parallel to and near the membrane is proportional to the perpendicular distance from the membrane [$u=(\tau/\mu)y$]. The first assumption is unlikely since the substrate material (starch) is a known foulant of membrane surfaces. Further, the enzyme itself is a material that is separated industrially by membranes and, in that context, is known to retard membrane flux according to gel models. However, the action of the enzyme is to consume the substrate, thus probably mitigating the expected tendency to foul the membrane. A responsive model for the flux must await the disclosure of experiments. The second assumption is valid only within a very thin layer on the membrane surface. The non-dimensional equations which govern the diffusion and production of the species are as follows:

$$Y\frac{\partial C_i}{\partial X} - \frac{\partial C_i}{\partial Y} = \frac{D_i \partial^2 C_i}{D_1 \partial Y^2} + S_{0,i} \qquad i=1,2,3 \qquad (10)$$

with the following boundary and initial conditions:

$$-\gamma_i C_{i,w} = \frac{D_i \partial C_i}{D_1 \partial Y})_{wall} \qquad (11a)$$

$$C_i(0,Y) = 1 \qquad (11b)$$

$$C_i(X, Y_{max}) = 1. \qquad (11c)$$

Here the subscript i=1 refers to the substrate, i=2

refers to the catalyst and i=3 refers to the product. The $S_{o,i}$ refers to the source term for each species. Thus the source term for different species can be expressed as follows:

$$S_{0,1} = -\frac{p_1}{CR}\frac{C_2C_1}{KR + C_1} \tag{12}$$

$$S_{0,2} = 0. \tag{13}$$

and

$$S_{0,3} = \left(\frac{p_1}{MWR}\right)\left(\frac{C_{2,b}}{C_{3,b}}\right)\frac{C_2C_1}{(KR + C_1)} \tag{14}$$

In these equations the concentrations $C_1$, $C_2$, and $C_3$ refer to non- dimensional concentration values. X and Y are non-dimensional distances along and perpendicular to the membrane surface respectively, whereas x and y refer to corresponding distances in meters. Other non-dimensional parameters used in these equations are:

$$\frac{D_1k_1}{J^2} \times MWR = p1, \tag{15a}$$

$$\frac{c_1(b)}{c_2(b)} = CR, \tag{15b}$$

$$\frac{k_m}{c_1(b)} = KR, \tag{15c}$$

$$X = \frac{xJ^3}{D_1^2}\left(\frac{\mu}{\tau}\right), \quad Y = \frac{Jy}{D_1}. \tag{15d}$$

where $(\mu/\tau)$ refer to the ratio of the viscosity and shear stress of the solution flowing through the membrane reactor.

The equations for concentration of species have been solved by writing a set of finite difference

equations and solving by an implicit scheme (11). Since the equations are coupled an iterative approach was necessary at each step.

To determine the production, an integral of the sugar component over the control volume stretching from the start to the end of the membrane channel and from the permeate side of membrane past the edge of all diffusion is considered. The production of sugar in this volume will be registered in the increase in sugar flux in the outflow versus that in the inflow sections. So the production P per unit thickness is given by:

$$P=\int_{X=0}^{X=X_{max}} [C_3(X,0)-C_3(X,Y_{max})]JdX+\int_{Y=0}^{Y=Y_{max}} [C_3(X_{max,Y})-C_3(0,Y)]\left(\frac{\tau}{\mu}\right)YdY \qquad (16)$$

The volume above would have production if all concentrations were non-polarized, that is, if all concentrations were at bulk conditions. There would be the same gradual increase in sugar in the mainstream of the flow, and the exit flow at the position corresponding to the end of the membrane would be elevated in sugar, reflecting the production. The careful reader will observe that the membrane reactor having the same concentration of enzyme in the bulk stream has more mass of enzyme than the equivalent volume in the imaged uniform state reactor. We have compensated for this by adjusting the concentration of enzyme for the uniform state reactor to make the mass of enzyme correspond exactly to the compared membrane reactor.

By subtracting the production for the uniform state reactor from the membrane reactor, the residual is the

excess production for the membrane reactor. The equation for excess production, $P-P_{u.s}$ per unit thickness of the membrane surface is:

$$P-P_{u.s}=\int_{X=0}^{X=X_{max}} [C_3(X,0)-C_{3,b}(X,0)]JdX+ \int_{Y=0}^{Y=Y_{max}} [C_3(X_{max,Y})-C_{3,b}(X_{max,Y})]\left(\frac{\tau}{\mu}\right)YdY + \frac{k_1}{1+KR}\left\{X_{max}Y_{max}-\frac{JX_{max}^2}{2U_\infty}\right\}(C_{2,b}-C_{2,b}^{mixed}) \quad (17)$$

The overall excess production due to concentration polarization as predicted by Datta (10) for the study herein is summarized in the following table. The length of the membrane reactor was assumed to be one meter in all the cases. It should be emphasized that the optimum reactor length for maximum excess production was not reached in all of the cases as the design variables used in this study puts the optimum reactor length greater than the one meter assumed here.

| % Production in Membrane Reactor | CR | Excess Production ÷ Total Production (%) | Excess Production (Kg/day) |
|---|---|---|---|
| 1.0 | 0.303 | 0.0577 | 5.48 |
| 1.5 | 0.202 | 0.039 | 5.57 |
| 2.0 | 0.1515 | 0.0302 | 5.75 |
| 2.5 | 0.1212 | 0.0246 | 5.84 |

Studies of the excess production per unit area are not very instructive without an example such as the one presented here. The fact of excess production and the observation of best length, serve to indicate that the conditions are potentially important. The relative reaction in uniform concentration in the reactor fluid

is certainly expected to be proportional to the volume per area of the reactor, so the excess production benefit will aid membrane reactors having thin channels over those in thick ones.

## CONCLUSIONS

A pair of reactors, one of conventional continuous design coupled with a subsequent membrane reactor, has been proposed. The majority product of the reaction is filtered and the reactor is subject to automatic process control by variation of the enzyme added between the devices. The reject stream, perhaps 4% of the feed to the process, will contain all of the "mud solids", the residual substrate for the reaction, essentially all of the enzyme, those partially reacted components not able to pass the membrane, and a volumetric proportion of the product. Because the product loss of the order of 4% will be excessive, the reject will require processing not described in this paper of a "washing" or "diafiltration" type, allowing the separation of desired product from undesired components. The enzyme use is not elevated or at least not significantly elevated above that required in a conventional operation.

The value of the reaction in the second, membrane, portion will depend directly on the volume per unit area characteristic of the membrane flow channels. It was seen that the residence time in the reactor is proportional to this index. Further, if the control concept is to be of real value, the calculations herein indicate that very thin channels (0.001 $m^3/m^2$) will have so little residence time that the reaction therein is of little use. Reactors with moderate channels (0.01 $m^3/m^2$) will provide conditions where a meaningful amount

of reaction, sufficient to form the basis for control of the product, can be expected.

The range of anticipated parameters for the uniform-state design of the reactor system described has been used to establish the actual conditions appropriate to evaluate the enhancement of reaction due to polarization in the membrane reactor. Such enhancement is due to the fact that the enzyme and substrate are rejected by the membrane and thus have elevated concentrations in a thin zone near the membrane. In this thin zone the reactions potentially occur at rates many times faster than in the bulk state. In the case evaluated herein the rate of reaction per unit volume in some locations was actually 10000 times as large as in the bulk. For very thin passages the fraction by which the reaction could be enhanced is increased. But the utility of the control function of the second stage would be necessarily lost as per the conclusion above.

The reaction advantage detailed above is located in a region of only a few micrometers in thickness and comprises a small portion of the entire reactor volume. The result is that the enhancement with volume per unit area of 0.01 $m^3/m^2$ predicts to be of negligible consequence for the conditions evaluated. Even with channels an order of magnitude thinner the advantage is minor. Thus, the simple uniform-state calculation of reaction in the membrane reactor will be a valid estimate for the rate of production in the case evaluated.



## NOMENCLATURE

$A_c$ = cross section of the continuous flow reactor.

$A_m$ = area of the membrane.

$\beta$ = ratio of shear stress to dynamic viscosity, $=\tau/\mu$.

$C_{1,0}$ = substrate feed concentration to the overall reactor (mass/volume).

$C_{1,m}$ = equilibrium substrate concentration in the membrane reactor (mass/volume).

$C_{2,b}^{mixed}$ = non-dimensional enzyme concentration in the bulk phase when the excess enzyme due to concentration polarization is mixed homogeneously in the solution.

$C_{3,f}$ = product feed concentration in the membrane reactor (mass/volume).

$C_{3full}$ = maximum possible product feed concentration in the membrane reactor (mass/volume).

$C_i$ = non-dimensional concentration of species i $(c_i/c_{i,b})$.

$c_i$ = concentration of species i (mass/volume).

$C_{i,b}$ = non-dimensional bulk concentration of species i.

$c_{i,b}$ = bulk concentration of species i.

$C_{i,w}$ = wall concentration of species i.

CR = concentration ratio between substrate and enzyme $(c_{1,b}/c_{2,b})$.

CR2 = concentration ratio between enzyme and product $(c_{2,b}/c_{3,b})$.

$D_i$ = molecular diffusivity of solute specie i.

DR = diffusion ratio between enzyme and substrate $(D_2/D_1)$.

$\dot{E}$ = enzyme requirement in volume per day.

$\gamma_i$ = rejection coefficient of species i.

$i$ = subscript defining different species, 1=substrate, 2=enzyme, 3=product.

$J$ = permeate flux through membrane.

$k_1$ = rate constant of Michaelis-Menten equation.

$K_m$ = inhibition index of Michaelis-Menten equation.

KR = non-dimensional inhibition index ($K_m/c_{1,b}$).

$\lambda$ = $k_1$*MWR/$K_m$

$\mu$ = dynamic viscosity of feed solution.

$MW_1$ = molecular weight of substrate.

MWR = molecular weight ratio between product and substrate.

$n$ = grid number.

$n_i$ = number of moles of solute i in solution.

$P$ = total rate of production in membrane reactor.

p1 = non-dimensional rate parameter of M-M equation ($D_1 k_1 MWR/J^2$).

$P_{u.s}$ = uniform state production in membrane reactor.

RF = rejection factor.

RR = rejection ratio between enzyme and substrate ($\gamma_2/\gamma_1$).

$S_{o,i}$ = source term for species i.

$\tau$ = shear stress at wall of the membrane.

$u$ = longitudinal velocity along membrane ($\tau y/\mu$).

$U_\infty$ = cross-flow velocity of the membrane reactor.

$v$ = transverse velocity across membrane (=- J).

$X$ = non-dimensional distance along membrane ($xJ^3\mu/\tau D_1{}^2$).

$x$ = coordinate denoting distance along membrane.

$X_{max}$ = corresponding non dimensional length of $x_{max}$.

$x_{max}$ = length of the reactor.

$Y$ = non-dimensional distance away from the membrane wall ($Jy/D_1$).

$y$ = coordinate denoting distance perpendicular to the membrane surface and away from it.

$Y_{max}$ = non dimensional Y at which $\partial C_i/\partial Y=0$.

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